

Second Moment Acceleration for Incident Fluxes on Cell Surfaces



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SWEEP-LESS TRANSPORT

Start with steady, isotropic, gray, radiation transport equation written in operator form. To solve iteratively we often lag the scattering source resulting in the source iteration:

$$\mathbf{L} \psi^{(l)} = \mathbf{S} \psi^{(l-1)} + q$$

The incident angular flux on the surface of each cell may also be lagged, giving the one cell inversion [1]

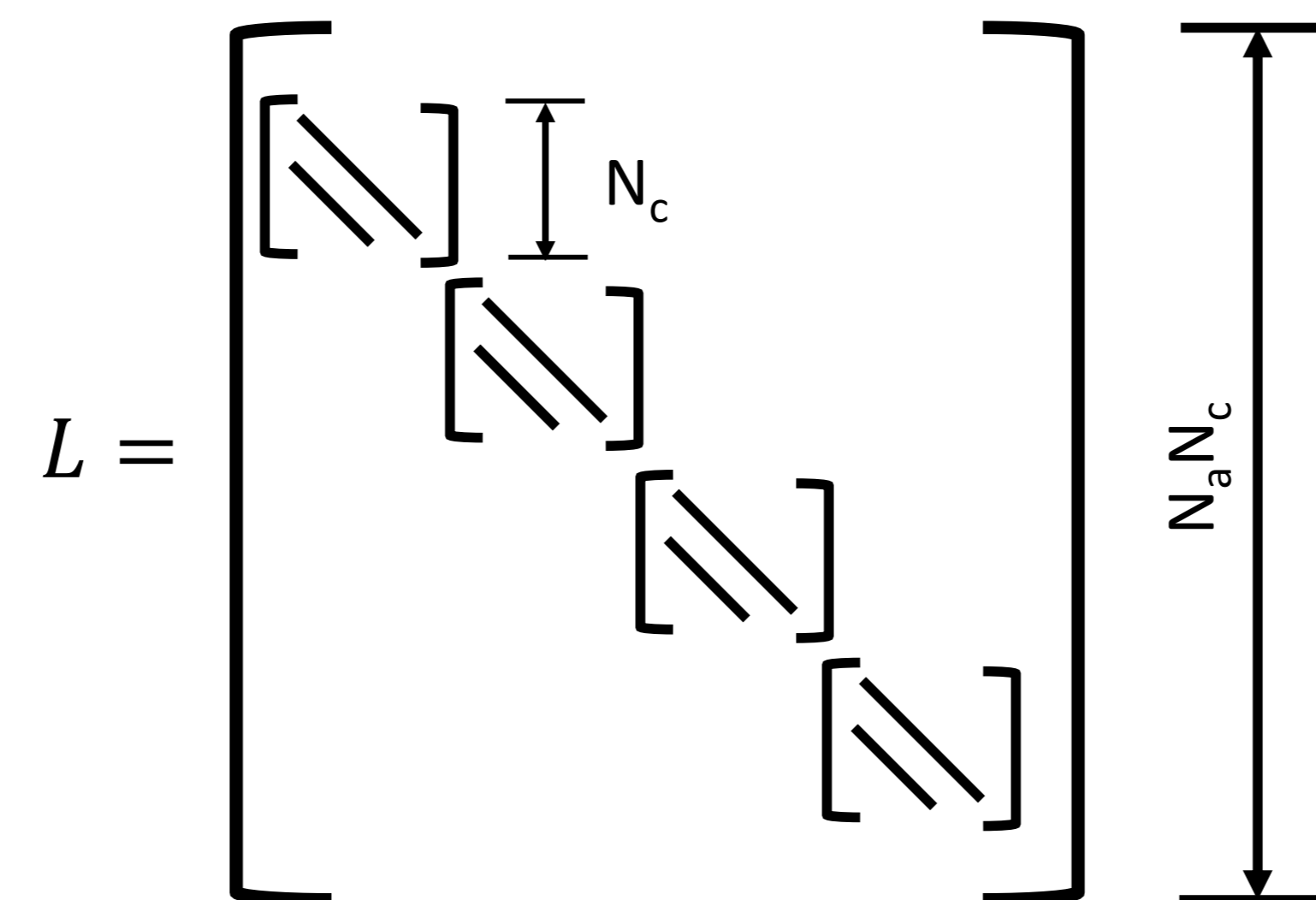
$$(\mathbf{L}_c - \mathbf{S}) \psi^{(l)} = (-\mathbf{L}_b) \psi^{(l-1)} + q$$

Where:

- ψ is angular flux (with associated angular moments: ϕ_0, ϕ_1, \dots)
- $\mathbf{L} = \mathbf{L}_b + \mathbf{L}_c$
 - \mathbf{L}_b within cell DOFs
 - \mathbf{L}_c cell bound DOFs
- \mathbf{S} is discrete to moment operator
- q isotopic material source
- l is the iteration index

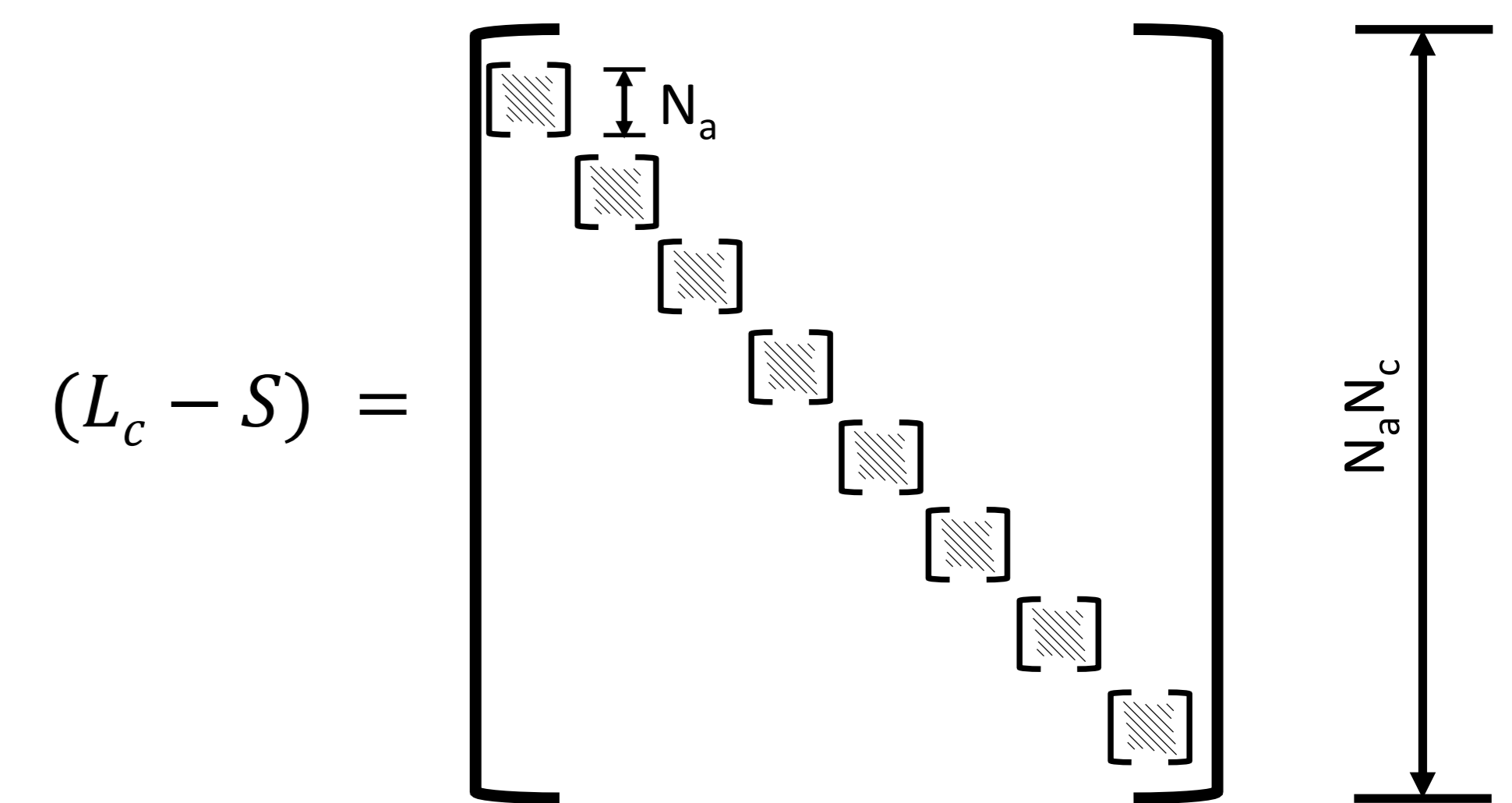
Transport Sweeps

\mathbf{L} (shown below) is sparse, block diagonal with the number of sparse lower triangular blocks equaling the number of angles (N_a). To efficiently invert \mathbf{L} use a transport sweep which is parallel over angle and memory efficient.

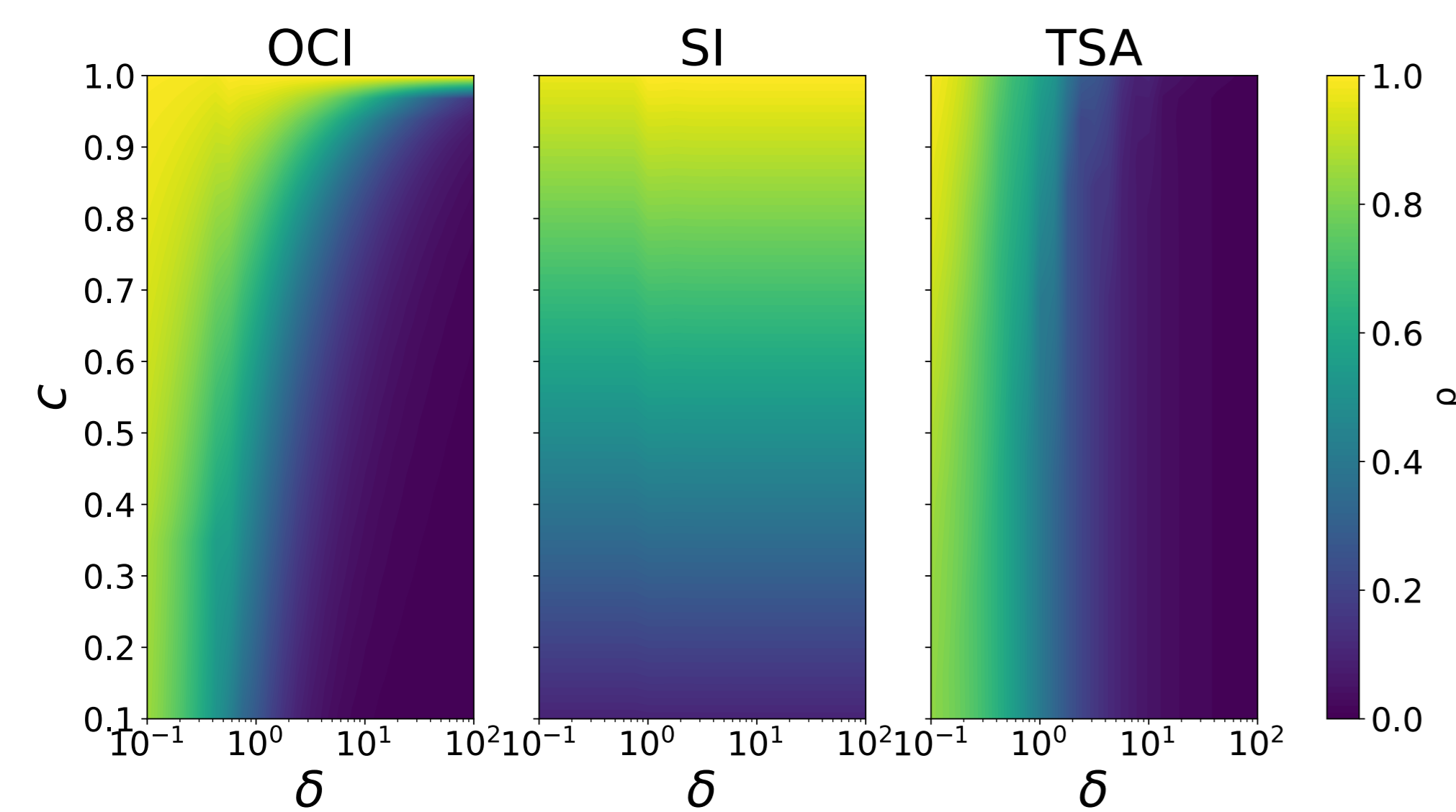


One Cell Inversion

$\mathbf{L}_c - \mathbf{S}$ (shown below) is also sparse block diagonal but with the number of dense systems equaling the number of cells (N_c). To invert, solve N_c number of dense independent systems. Parallel over the number of cells, efficient on GPUs [1] [3].



CONVERGENCE RATE



The Problem

Spectral radius of one cell inversion goes to unity in the scattering & thin limits [1] [2] [3]. This has been found in and is often described as asynchronicity in space.

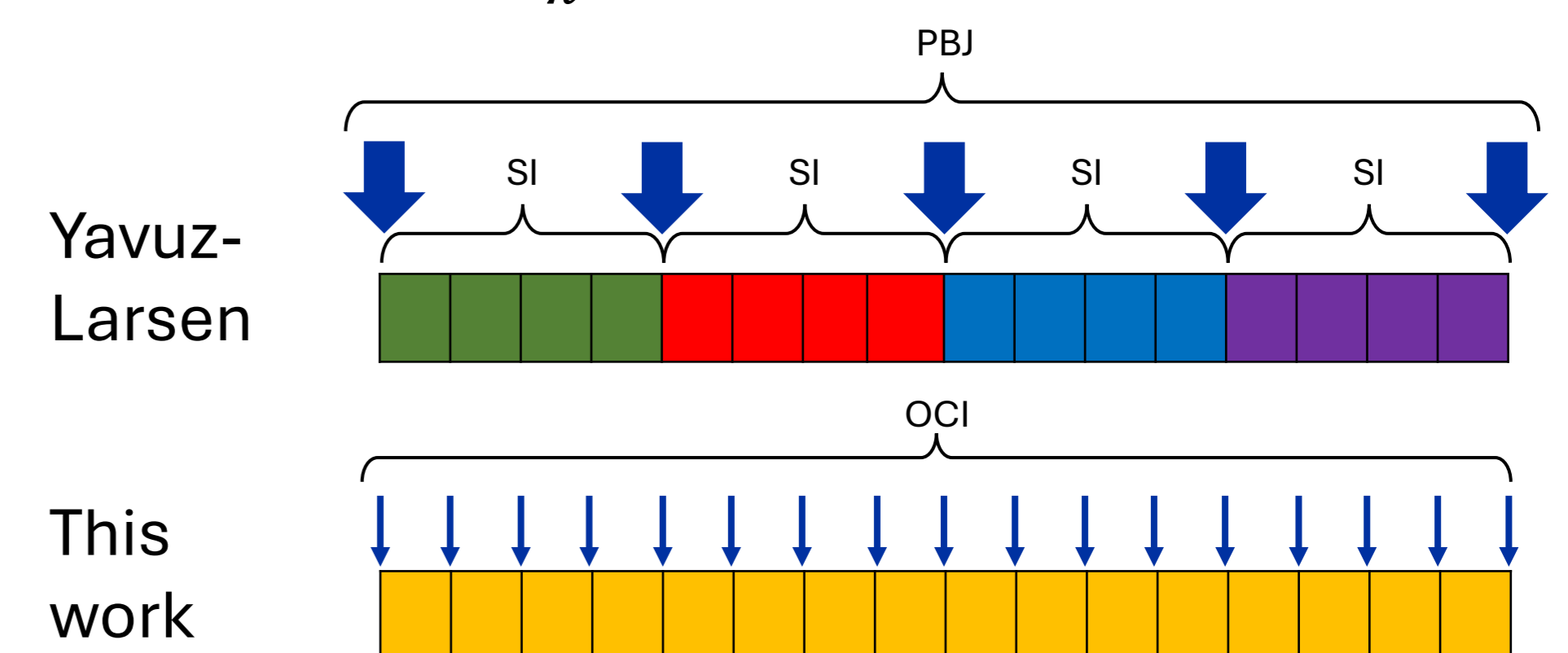
Transport synthetic acceleration (TSA) has been shown to ameliorate these issues but, itself requires a transport sweep [3]. Time dependence has also been shown to accelerate the problem due to increasing optical thickness [1].

Is there a preconditioner for steady problems in the thin and/or diffusive limits that does not require a sweep?

A Solution?

Yavuz-like update [2] on the surface of each cell

$$\psi^{(l+1)} = \psi^{(l+1/2)} + \frac{1}{2\pi} (\phi_0^{(l+3/4)} - \phi_0^{(l+1/2)}) + \frac{1}{\pi} \Omega \cdot (\phi_1^{(l+3/4)} - \phi_1^{(l+1/2)})$$

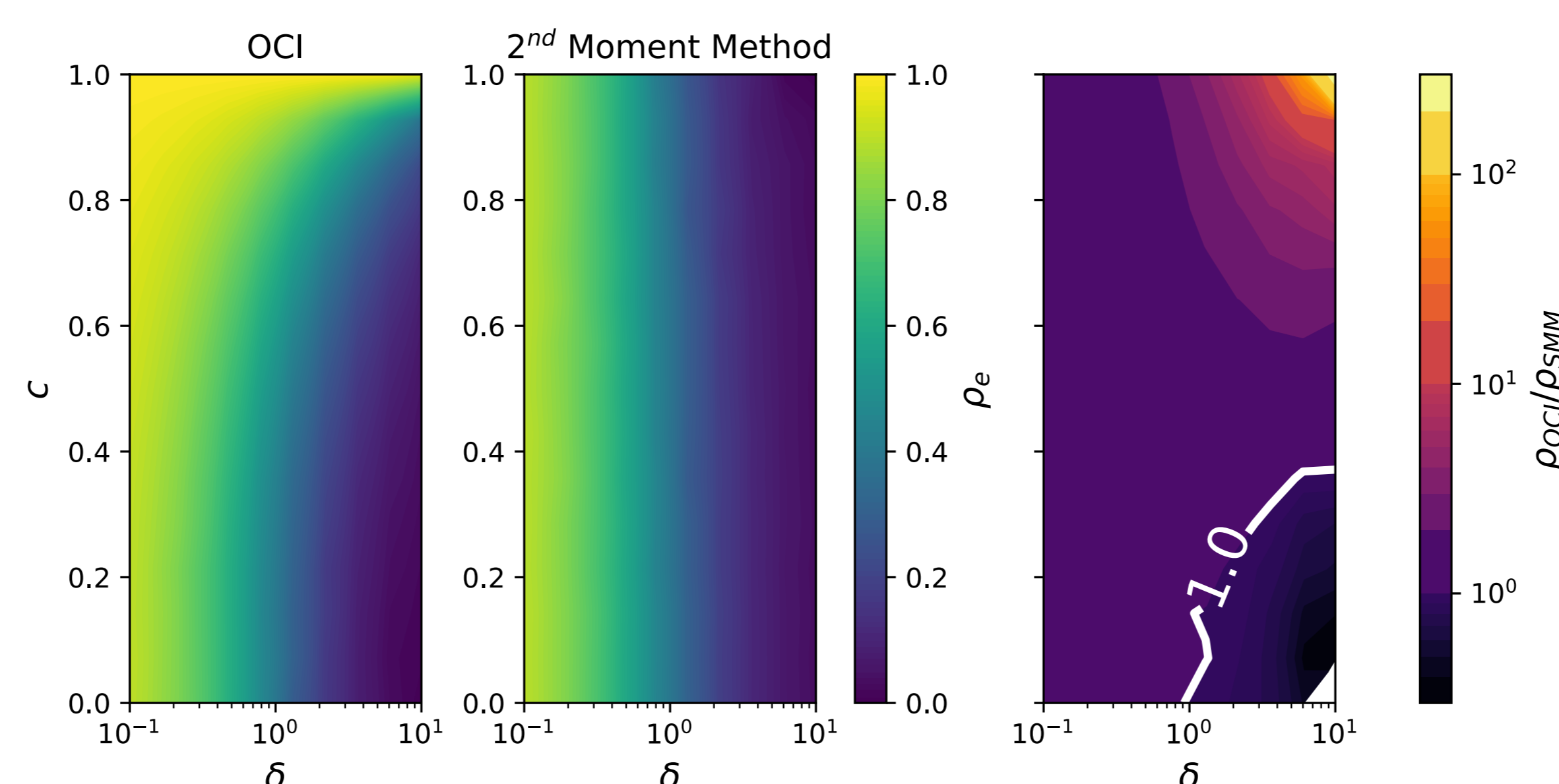


RESULTS

$l+3/4$ moments provided from a semi-consistent second moment method derived using Adams-Martin modified 4-step procedure [4].

Estimations of spectral radius provided from ratio of subsequent residuals when solving a 1D large homogeneous medium problem are shown at right. These results seem to indicate speedup especially in the diffusive limit and little-to-no acceleration in the optically thin limit.

Significant effort to extend this method to 2-spatial dimensions has yet to produced an operable method except with GMRES and/or damping for inconsistent or consistent derivations of second moment equations.



CONCLUSIONS

While this Yavuz update with a second moment method seems to converge the thick diffusive limit for 1D problems it does not dramatically support convergence in the thin limit.

The slowest converging error modes for OCI are spatially global. A preconditioner to converge this likely requires some kind of global information to be propagated.

A preconditioner in the diffusive limit for OCI iterations remains elusive.

Citations

- [1] J. P. Morgan, I. Variansyah, T. Palmer, and K. Niemeier. Nucl. Sci. Eng. 200:5, 1074–1093 (2026).
- [2] M. Yavuz and E. W. Larsen, Transp. Theory Stat. Phys. 18:2, 205-219 (1989).
- [3] M. Rosa and J. Warsa, Trans. Am. Nucl. Soc., 101, 1103-1107 (2009).
- [4] J. P. Morgan, PhD Thesis, Oregon State University (2025).