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Second Moment Acceleration for Incident Fluxes on Cell Surfaces*

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The steady, one-group, isotropic, discrete ordinates (S_N) neutron transport equation in operator notation is,

$$L\psi = S\psi + q \;, \tag{1}$$

where ψ is the angular flux, L is the advection–reaction operator, q is a material source, and S is the scattering operator, subject to boundary conditions. Lagging $S\psi$, and solving the resulting system in a matrix-free manner (via sweeps) Eq (1) is the "source iteration"—a preconditioned fixed-point Richardson iteration. L can be further decomposed to $L = L_c + L_b$, which are the within cell and boundary contributions respectively to the advection–reaction equation. Allowing the angular flux on the surface of the cell to lag and solving in terms of the scattering and within cell advection–reaction components gives,

$$(L_{c,i} - S_i)\psi_i^{(l+1)} = (-L_{b,i})\,\psi^{(l)} + q \;, \tag{2}$$

for each cell $i = 1, \ldots, N$. This is the one cell inversion iteration or the cell-wise block Jacobi iteration. Eq (2) can be solved spatially parallel, presenting as small, dense, linear algebra systems which solved efficiently on GPUs [1]. Convergence for Eq (2) now depends on both scattering ratio and cellular optical thickness, going to unity in the diffusive and thin-mesh limit.

Inspired by Yavuz and Larsen [2] we develop a 2D second moment method on Cartesian grids and update incident angular fluxes on the surface of each cell by

$$\psi^{(l+1)} = \psi^{(l+1/2)} + \frac{1}{2\pi}\left(\phi_0^{(l+3/4)} - \phi_0^{(l+1/2)}\right) + \frac{1}{\pi}\Omega\cdot\left(\phi_1^{(l+3/4)} - \phi_1^{(l+1/2)}\right) \;, \tag{3}$$

in continuous 2D, where ϕ_j is the j^{th} angular moment of ψ. The $(l+1/2)$ terms are from an OCI transport solution and $(l+3/4)$ terms are from an inconsistent second moment method. Eq (3) shows modest acceleration in the scattering regime. We also implement an update using a half-range P_1 expansion, similar to Eq (3), on cell surfaces for incident ψ. With this update, eliminating the scattering operator from the transport step, $(l+1/2)$, had little impact to the converged solution. However, these results are preliminary.

References

[1] J. P. Morgan, I. Variansyah, T. S. Palmer, K. E. Niemeyer, Nucl. Sci. Eng., *in press* (2025).

[2] M. Yavuz and E. W. Larsen, Transp. Theory Stat. Phys. **18**:2, 205-219 (1989).

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